N Widths In Approximation Theory

This account of totally positive matrices treats their central properties with full proofs and a complete bibliography.

This is a translation of the fifth and final volume in a special cycle of publications in commemoration of the 50th anniversary of the Steklov Mathematical Institute of the Academy of Sciences in the USSR. The purpose of the special cycle was to present surveys of work on certain important trends and problems pursued at the Institute. Because the choice of the form and character of the surveys were left up to the authors, the surveys do not necessarily form a comprehensive overview, but rather represent the authors' perspectives on the important developments. The survey papers in this collection range over a variety of areas, including - probability theory and mathematical statistics, metric theory of functions, approximation of functions, descriptive set theory, spaces with an indefinite metric, group representations, mathematical problems of mechanics and spaces of functions of several real variables and some applications.

In recent years approximation theory and the theory of orthogonal polynomials have witnessed a dramatic increase in the number of solutions of difficult and previously untouchable problems. This is due to the interaction of approximation theoretical techniques with classical potential theory (more precisely, the theory of logarithmic potentials, which is directly related to polynomials and to problems in the plane or on

the real line). Most of the applications are based on an exten sion of classical logarithmic potential theory to the case when there is a weight (external field) present. The list of recent developments is quite impressive and includes: creation of the theory of non-classical orthogonal polynomials with re spect to exponential weights; the theory of orthogonal polynomials with respect to general measures with compact support; the theory of incomplete polynomials and their widespread generalizations, and the theory of multipoint Pade approximation. The new approach has produced long sought solutions for many problems; most notably, the Freud problems on the asymptotics of orthogonal polynomials with a respect to weights of the form exp(-Ixl); the "l/9-th" conjecture on rational approximation of exp(x); and the problem of the exact asymptotic constant in the rational approximation of Ixl. One aim of the present book is to provide a self-contained introduction to the aforementioned "weighted" potential theory as well as to its numerous applications. As a side-product we shall also fully develop the classical theory of logarithmic potentials.

An expert guide to the relationship between information theory and the physics of wave propagation, covering stochastic and deterministic approaches, engineering applications, and the universal physical limits of radiation. It is an ideal reference for researchers and graduate students in electrical engineering, physics, and applied mathematics.

This monograph describes advances in the theory of extremal problems in classes of

functions defined by a majorizing modulus of continuity w. In particular, an extensive account is given of structural, limiting, and extremal properties of perfect w-splines generalizing standard polynomial perfect splines in the theory of Sobolev classes. In this context special attention is paid to the qualitative description of Chebyshev w-splines and w-polynomials associated with the Kolmogorov problem of n-widths and sharp additive inequalities between the norms of intermediate derivatives in functional classes with a bounding modulus of continuity. Since, as a rule, the techniques of the theory of Sobolev classes are inapplicable in such classes, novel geometrical methods are developed based on entirely new ideas. The book can be used profitably by pure or applied scientists looking for mathematical approaches to the solution of practical problems for which standard methods do not work. The scope of problems treated in the monograph, ranging from the maximization of integral functionals, characterization of the structure of equimeasurable functions, construction of Chebyshev splines through applications of fixed point theorems to the solution of integral equations related to the classical Euler equation, appeals to mathematicians specializing in approximation theory, functional and convex analysis, optimization, topology, and integral equations . Volume 190, issue 1 of 4. Translated from the Russian. Comprises materials of the All-Union School on the Theory of Functions (October 1987, Amberd), presenting papers on the coefficients of cosine series with nonnegative partial sums, bases in function spaces and the Franklin system, vectors with c

This ENCYCLOPAEDIA OF MATHEMATICS aims to be a reference work for all parts of mathe matics. It is a translation with updates and editorial comments of the Soviet Mathematical Encyclopaedia published by 'Soviet Encyclopaedia Publishing House' in five volumes in 1977-1985. The annotated translation consists of ten volumes including a special index volume. There are three kinds of articles in this ENCYCLOPAEDIA. First of all there are survey-type articles dealing with the various main directions in mathematics (where a rather fine subdivi sion has been used). The main requirement for these articles has been that they should give a reasonably complete up-to-date account of the current state of affairs in these areas and that they should be maximally accessible. On the whole, these articles should be understandable to mathematics students in their first specialization years, to graduates from other mathematical areas and, depending on the specific subject, to specialists in other domains of science, en gineers and teachers of mathematics. These articles treat their material at a fairly general level and aim to give an idea of the kind of problems, techniques and concepts involved in the area in question. They also contain background and motivation rather than precise statements of precise theorems with detailed definitions and technical details on how to carry out proofs and constructions. The second kind of article, of medium length, contains more detailed concrete problems, results and techniques. This book constitutes the refereed post-proceedings of the Second International Conference on Theoretical and Mathematical Foundations of Computer Science,

ICTMF 2011, held in Singapore in May 2011. The conference was held together with the Second International Conference on High Performance Networking, Computing, and Communication systems, ICHCC 2011, which proceedings are published in CCIS 163. The 84 revised selected papers presented were carefully reviewed and selected for inclusion in the book. The topics covered range from computational science, engineering and technology to digital signal processing, and computational biology to game theory, and other related topices.

This volume consists of 24 refereed carefully edited papers on various topics in multivariate approximation. It represents the proceedings of a workshop organized by the University of Firenze, and held in September 1995 in Montecatini, Italy.The main themes of the volume are multiresolution analysis and wavelets, multidimensional interpolation and smoothing, and computer-aided geometric design. A number of particular topics are included, like subdivision algorithms, constrained approximation and shape-preserving algorithms, thin plate splines, radial basis functions, treatment of scattered data, rational surfaces and offsets, blossoming, grid generation, surface reconstruction, algebraic curves and surfaces, and neural networks.

This book is devoted to some topical problems and applications of operator theory and its interplay with modern complex analysis. It consists of 20 selected survey papers that represent updated (mainly plenary) addresses to the IWOTA 2000 conference held at Bordeaux from June 13 to 16, 2000. The main subjects of the volume include: - spectral

analysis of periodic differential operators and delay equations, stabilizing controllers, Fourier multipliers; - multivariable operator theory, model theory, commutant lifting theorems, coisometric realizations; - Hankel operators and forms; - operator algebras; - the Bellman function approach in singular integrals and harmonic analysis, singular integral operators and integral representations; - approximation in holomorphic spaces. These subjects are unified by the common "operator theoretic approach" and the systematic use of modern function theory techniques.

The papers in this book, first presented at a 1986 AMS Short Course, give a brief introduction to approximation theory and some of its current areas of active research, both theoretical and applied. The first lecture describes and illustrates the basic concerns of the field. Topics highlighted in the other lectures include the following: approximation in the complex domain, N-width, optimal recovery, interpolation, algorithms for approximation, and splines, with a strong emphasis on a multivariate setting for the last three topics. The book is aimed at mathematicians interested in an introduction to areas of current research and to engineers and scientists interested in exploring the field for possible applications to their own fields. The book is best understood by those with a standard first graduate course in real and complex analysis, but some of the presentations are accessible with the minimal requirements of advanced calculus and linear algebra.

Designed to give a contemporary international survey of research activities in

approximation theory and special functions, this book brings together the work of approximation theorists from North America, Western Europe, Asia, Russia, the Ukraine, and several other former Soviet countries. Contents include: results dealing with q-hypergeometric functions, differencehypergeometric functions and basic hypergeometric series with Schur function argument; the theory of orthogonal polynomials and expansions, including generalizations of Szegö type asymptotics and connections with Jacobi matrices; the convergence theory for Padé and Hermite-Padé approximants, with emphasis on techniques from potential theory; material on wavelets and fractals and their relationship to invariant measures and nonlinear approximation; generalizations of de Brange's in equality for univalent functions in a quasi-orthogonal Hilbert space setting; applications of results concerning approximation by entire functions and the problem of analytic continuation; and other topics.

These proceedings are based on papers presented at the international conference Approximation Theory XV, which was held May 22–25, 2016 in San Antonio, Texas. The conference was the fifteenth in a series of meetings in Approximation Theory held at various locations in the United States, and was attended by 146 participants. The book contains longer survey papers by some of the invited speakers covering topics such as compressive sensing, isogeometric analysis, and scaling limits of polynomials and entire functions of exponential type. The book also includes papers on a variety of current topics in Approximation Theory drawn from areas such as advances in kernel approximation with applications, approximation theory and algebraic geometry, multivariate splines for applications, practical function

approximation, approximation of PDEs, wavelets and framelets with applications, approximation theory in signal processing, compressive sensing, rational interpolation, spline approximation in isogeometric analysis, approximation of fractional differential equations, numerical integration formulas, and trigonometric polynomial approximation.

' This is the collection of the refereed and edited papers presented at the 8th Texas International Conference on Approximation Theory. It is interdisciplinary in nature and consists of two volumes. The central theme of Vol. I is the core of approximation theory. It includes such important areas as qualitative approximations, interpolation theory, rational approximations, radial-basis functions, and splines. The second volume focuses on topics related to wavelet analysis, including multiresolution and multi-level approximation, subdivision schemes in CAGD, and applications. Contents:Volume I:Differentiated Shift-Invariant Integral Operators (G A Anastassiou)Efficient Matrix Methods for the True Least-Squares Approximation of Structured Multivariate Data (I J Anderson & J C Mason)Vectorially Minimal Projections (A Bacopoulos & B L Chalmers)Error of an Arbitrary Order for the Approximate Solution of Systems of nth Order Differential Equations with Spline Functions (B S Badr et al)A Note on Irving Glicksberg"s Pseudocompactness Papers (J Blatter & H König)A Multivariate Divided Difference (C de Boor)Approximation Using Positive Definite Functions (E W Cheney)A Brief Glance at the Research of Ward Cheney (W Light)Ideas of Weighted Polynomial Approximation on (-?,?) (D S Lubinsky)Piecewise Convex Function Estimation and Model Selection (K S Riedel)Multivariate Interpolation and Approximation by Translates of a Basis Function (R Schaback)and other papersVolume II:A Wavelet-Like Unconditional Basis (K–F Chang)Multivariate Interpolating Wavelets (C K Chui & C Li)Nonlinear Wavelet Approximation

and Image Compression (A Cohen)Wavelets and Interactive Surface Modeling (E Cornea et al)Multiscale Analysis, Approximation, and the Interpolation Spaces (W Dahmen)Using Fredholm Determinants to Estimate the Smoothness of Refinable Functions (I Daubechies)Stability and Independence of the Shifts of a Multivariate Refinable Function (T Hogan)Refinable Shift-Invariant Spaces: From Splines to Wavelets (R Q Jia)Weakly Singular Fredholm Integral Equations I: Singularity Preserving Wavelet-Galerkin Methods (C A Micchelli & Y–S Xu)and other papers Readership: Applied mathematicians.
Keywords:Proceedings;Conference;Approximation Theory;College Station, TX (USA);Interpolation;Wavelets;Multilevel Approximation'

These Proceedings include 42 of the 49 invited conference papers, three papers sub mitted subsequently, and a report devoted to new and unsolved problems based on two special problem sessions and as augmented by later communications from the participants. In addition, there are four short accounts that emphasize the personality of the scholars to whom the proceedings are dedicated. Due to the large number of contributors, the length of the papers had to be restricted. This volume is again devoted to recent significant results obtained in approximation theory, harmonic analysis, functional analysis, and operator theory. The papers solicited include in addition survey articles that not only describe fundamental advances in their subfields, but many also emphasize basic interconnections between the various research areas. They tend to reflect the range of interests of the organizers and of their immediate colleagues and collaborators. The papers have been grouped according to subject matter into ten chapters. Chap ter I, on operator theory, is devoted to certain classes of operators such as contraction, hyponormal, and accretive operators, as well as to

suboperators and semi groups of operators. Chapter II, on functional analysis, contains papers on function spaces, algebras, ideals, and generalized functions. Chapter III, on abstract approximation, is concerned with the comparison of approximation processes, the gliding hump method, certain inter polation spaces, and n-widths.

The works of George G. Lorentz, spanning more than 60 years, have played a significant role in the development and evolution of mathematical analysis. The papers presented in this volume represent a selection of his best works, along with commentary from his students and colleagues.

The approximation of functions of several variables continues to be a difficult problem in scientific computing because many of the algorithms required for such problems have yet to be written. This monograph is written for a broad audience of computational mathematicians and statisticians concerned with the development of algorithms or the derivation of approximations from linear projections, of which the interpolating operators are an important example. As an aid to both researchers and students, a bibliography of more than 200 titles is included.

My original introduction to this subject was through conservations, and ultimate ly joint work with C. A. Micchelli. I am grateful to him and to Profs. C. de Boor, E. W. Cheney, S. D. Fisher and A. A. Melkman who read various portions of the manuscript and whose suggestions were most helpful. Errors in accuracy and omissions are totally my responsibility. I would like to express my appreciation to the SERC of Great Britain and to the Department of Mathematics of the University of Lancaster for the year spent there during which large portions of the manuscript were written, and also to the European Research Office of the U.S. Army for its financial support of my research endeavors. Thanks are also due to Marion Marks who typed

portions of the manuscript. Haifa, 1984 Allan Pinkus Table of Contents 1 Chapter I. Introduction . . . . . Chapter II. Basic Properties of n-Widths . 9 1. Properties of d • • • • • • • • • • 9 n 15 2. Existence of Optimal Subspaces for d • n n 17 3. Properties of d • • • • • • 20 4. Properties of b • • • • • • n 5. Inequalities Between n-Widths 22 n 6. Duality Between d and d • • 27 n 7. n-Widths of Mappings of the Unit Ball 29 8. Some Relationships Between dn(T), dn(T) and bn(T) . 32 37 Notes and References . . . . . .

This textbook is designed for graduate students in mathematics, physics, engineering, and computer science. Its purpose is to guide the reader in exploring contemporary approximation theory. The emphasis is on multi-variable approximation theory, i.e., the approximation of functions in several variables, as opposed to the classical theory of functions in one variable. Most of the topics in the book, heretofore accessible only through research papers, are treated here from the basics to the currently active research, often motivated by practical problems arising in diverse applications such as science, engineering, geophysics, and business and economics. Among these topics are projections, interpolation paradigms, positive definite functions, interpolation theorems of Schoenberg and Micchelli, tomography, artificial neural networks, wavelets, thin-plate splines, box splines, ridge functions, and convolutions. An important and valuable feature of the book is the bibliography of almost 600 items directing the reader to important books and research papers. There are 438 problems and exercises scattered through the book allowing the student reader to get a better understanding of the subject.

These are the Proceedings of the NATO Advanced Study Institute on Approximation Theory, Spline Functions and Applications held in the Hotel villa del Mare, Maratea, Italy between April

28,1991 and May 9, 1991. The principal aim of the Advanced Study Institute, as reflected in these Proceedings, was to bring together recent and up-to-date developments of the subject, and to give directions for future research. Amongst the main topics covered during this Advanced Study Institute is the subject of uni variate and multivariate wavelet decomposition over spline spaces. This is a relatively new area in approximation theory and an increasingly impor tant subject. The work involves key techniques in approximation theory cardinal splines, B-splines, Euler-Frobenius polynomials, spline spaces with non-uniform knot sequences. A number of scientific applications are also highlighted, most notably applications to signal processing and digital im age processing. Developments in the area of approximation of functions examined in the course of our discussions include approximation of periodic phenomena over irregular node distributions, scattered data interpolation, Pade approximants in one and several variables, approximation properties of weighted Chebyshev polynomials, minimax approximations, and the Strang Fix conditions and their relation to radial functions. I express my sincere thanks to the members of the Advisory Commit tee, Professors B. Beauzamy, E. W. Cheney, J. Meinguet, D. Roux, and G. M. Phillips. My sincere appreciation and thanks go to A. Carbone, E. DePas cale, R. Charron, and B.

The book incorporates research papers and surveys written by participants ofan International Scientific Programme on Approximation Theory jointly supervised by Institute for Constructive Mathematics of University of South Florida at Tampa, USA and the Euler International Mathematical Instituteat St. Petersburg, Russia. The aim of the Programme was to present new developments in Constructive Approximation Theory. The topics of the papers are: asymptotic behaviour of orthogonal polynomials, rational approximation of classical functions,

quadrature formulas, theory of n-widths, nonlinear approximation in Hardy algebras,numerical results on best polynomial approximations, wavelet analysis. FROM THE CONTENTS: E.A. Rakhmanov: Strong asymptotics for orthogonal polynomials associated with exponential weights on R.- A.L. Levin, E.B. Saff: Exact Convergence Rates for Best Lp Rational Approximation to the Signum Function and for Optimal Quadrature in Hp.- H. Stahl: Uniform Rational Approximation of x .- M. Rahman, S.K. Suslov: Classical Biorthogonal Rational Functions.- V.P. Havin, A. Presa Sague: Approximation properties of harmonic vector fields and differential forms.- O.G. Parfenov: Extremal problems for Blaschke products and N-widths.- A.J. Carpenter, R.S. Varga: Some Numerical Results on Best Uniform Polynomial Approximation of x on 0,1 .- J.S. Geronimo: Polynomials Orthogonal on the Unit Circle with Random Recurrence Coefficients.- S. Khrushchev: Parameters of orthogonal polynomials.- V.N. Temlyakov: The universality of the Fibonacci cubature formulas.

The papers in this volume were presented at an International Symposium on Optimal Estimation in Approximation Theory which was held in Freudenstadt, Federal Republic of Germany, September 27-29, 1976. The symposium was sponsored by the IBM World Trade Europe/Middle East/Africa Corporation, Paris, and IBM Germany. On behalf of all the participants we wish to express our appreciation to the spon sors for their generous support. In the past few years the quantification of the notion of com plexity for various important computational procedures (e. g. multi plication of numbers or matrices) has been widely studied. Some such concepts are necessary ingredients in the quest for optimal, or nearly optimal, algorithms. The purpose of this symposium was to present recent results of similar character in the field or ap proximation theory, as well as to describe the algorithms currently

being used in important areas of application of approximation theory such as: crystallography, data transmission systems, cartography, reconstruction from x-rays, planning of radiation treatment, optical perception, analysis of decay processes and inertial navigation system control. It was the hope of the organizers that this con frontation of theory and practice would be of benefit to both groups. Whatever success th•~ symposium had is due, in no small part, to the generous and wise scientific counsel of Professor Helmut Werner, to whom the organizers are most grateful. Dr. T. J. Rivlin Dr. P. Schweitzer IBM T. J. Watson Research Center IBM Germany Scientific and Education Programs Yorktown Heights, N. Y.

In Fourier Analysis and Approximation of Functions basics of classical Fourier Analysis are given as well as those of approximation by polynomials, splines and entire functions of exponential type. In Chapter 1 which has an introductory nature, theorems on convergence, in that or another sense, of integral operators are given. In Chapter 2 basic properties of simple and multiple Fourier series are discussed, while in Chapter 3 those of Fourier integrals are studied. The first three chapters as well as partially Chapter 4 and classical Wiener, Bochner, Bernstein, Khintchin, and Beurling theorems in Chapter 6 might be interesting and available to all familiar with fundamentals of integration theory and elements of Complex Analysis and Operator Theory. Applied mathematicians interested in harmonic analysis and/or numerical methods based on ideas of Approximation Theory are among them. In Chapters 6-11 very recent results are sometimes given in certain directions. Many of these results have never appeared as a book or certain consistent part of a book and can be found only in periodics; looking for them in numerous journals might be quite onerous, thus this book may work as a reference source. The methods used in the book are those of classical analysis, Fourier

Analysis in finite-dimensional Euclidean space Diophantine Analysis, and random choice.
This authoritative volume comprises the plenary lectures and articles by many of the field's leading researchers who were brought together for the fourth time at the congress of the International Society for Analysis, its Applications and Computation (ISAAC).A wide spectrum of topics in modern analysis is covered by the fully refereed contributions, such as complex analysis, nonlinear analysis, inverse problems, wavelets, signals and images. In particular, important areas — not given special emphasis in previous meetings — include special functions and orthogonal polynomials, harmonic analysis, and partial differential equations.
A self-contained introduction for non-specialists, or a reference work for experts, on the area of approximation theory concerned with exact constants.
Intended for a wide range of readers, this book covers the main ideas of convex analysis and approximation theory. The author discusses the sources of these two trends in mathematical analysis, develops the main concepts and results, and mentions some beautiful theorems. The relationship of convex analysis to optimization problems, to the calculus of variations, to optimal control and to geometry is considered, and the evolution of the ideas underlying approximation theory, from its origins to the present day, is discussed. The book is addressed both to students who want to acquaint themselves with these trends and to lecturers in mathematical analysis, optimization and numerical methods, as well as to researchers in these fields who would like to tackle the topic as a whole and seek inspiration for its further development.
In these notes different deterministic and stochastic error bounds of numerical analysis

are investigated. For many computational problems we have only partial information (such as n function values) and consequently they can only be solved with uncertainty in the answer. Optimal methods and optimal error bounds are sought if only the type of information is indicated. First, worst case error bounds and their relation to the theory of n-widths are considered; special problems such approximation, optimization, and integration for different function classes are studied and adaptive and nonadaptive methods are compared. Deterministic (worst case) error bounds are often unrealistic and should be complemented by different average error bounds. The error of Monte Carlo methods and the average error of deterministic methods are discussed as are the conceptual difficulties of different average errors. An appendix deals with the existence and uniqueness of optimal methods. This book is an introduction to the area and also a research monograph containing new results. It is addressd to a general mathematical audience as well as specialists in the areas of numerical analysis and approximation theory (especially optimal recovery and information-based complexity).
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