Elliptic Boundary Value Problems In The Spaces Of Distributions 1st Edition

A co-publication of the AMS and Centre de Recherches Mathématiques In this monograph the authors study the well-posedness of boundary value problems of Dirichlet and Neumann type for elliptic systems on the upper half-space with coefficients independent of the transversal variable and with boundary data in fractional Hardy–Sobolev and Besov spaces. The authors use the so-called “first order approach” which uses minimal assumptions on the coefficients and thus allows for complex coefficients and for systems of equations. This self-contained exposition of the first order approach offers new results with detailed proofs in a clear and accessible way and will become a valuable reference for graduate students and researchers working in partial differential equations and harmonic analysis.

Derived from a lecture series for college mathematics students, introduces the methods of dealing with elliptical boundary-value problems--both the theory and the numerical analysis. Includes exercises. Translated and somewhat expanded from the 1987 German version. Annotation copyright by Book News, Inc., Portland, OR

A marriage of the finite-differences method with variational methods for solving boundary-value problems, the finite-element method is superior in many ways to finite-differences alone. This self-contained text for advanced undergraduates and graduate students is intended to imbed this combination of methods into the framework of functional analysis and to explain its applications to approximation of nonhomogeneous boundary-value problems for elliptic operators. The treatment begins with a summary of the main results established in the book. Chapter 1 introduces the variational method and the finite-difference method in the simple case of second-order differential equations. Chapters 2 and 3 concern abstract approximations of Hilbert spaces and linear operators, and Chapters 4 and 5 study finite-element approximations of Sobolev spaces. The remaining four chapters consider several methods for approximating nonhomogeneous boundary-value problems for elliptic operators.

This book describes state-of-the-art advances and applications of the unified transform and its relation to the boundary element method. The authors present the solution of boundary value problems from several different perspectives, in particular the type of problems modeled by partial differential equations (PDEs). They discuss recent applications of the unified transform to the analysis and numerical modeling of boundary value problems for linear and integrable nonlinear PDEs and the closely related boundary element method, a well-established numerical approach for solving linear elliptic PDEs.÷ The text is divided into three parts. Part I contains new theoretical results on linear and nonlinear evolutionary and elliptic problems. New explicit solution representations for several classes of boundary value problems are constructed and rigorously analyzed. Part II is a detailed overview of variational formulations for elliptic problems. It places the unified transform approach in a classic context alongside the boundary element method and stresses its novelty. Part III presents recent numerical applications based on the boundary element method and on the unified transform.

This monograph systematically treats a theory of elliptic boundary value problems in domains without singularities and in domains with conical or cuspidal points. This exposition is self-contained and a priori requires only basic knowledge of functional analysis. Restricting to boundary value problems formed by differential operators and avoiding the use of pseudo-differential operators makes the book accessible for a wider readership. The authors concentrate on fundamental results of the theory: estimates for solutions in different function spaces, the Fredholm property of the operator of the boundary value problem, regularity assertions and asymptotic formulas for the solutions near singular points. A special feature of the book is that the solutions of the boundary value problems are considered in Sobolev spaces of both positive and negative orders. Results of the general theory are illustrated by concrete examples. The book may be used for courses in partial differential equations.

The book contains a systematic treatment of the qualitative theory of elliptic boundary value problems for linear and quasilinear second order equations in non-smooth domains. The authors concentrate on the following fundamental results: sharp estimates for strong and weak solutions, solvability of the boundary value problems, regularity assertions for solutions near singular points. Key features: * New the Hardy – Friedrichs – Wirtinger type inequalities as well as new integral inequalities related to the Cauchy problem for a differential equation. * Precise exponents of the solution decreasing rate near boundary singular points and best possible conditions for this. * The question about the influence of the coefficients smoothness on the regularity of solutions. * New existence theorems for the Dirichlet problem for linear and quasilinear equations in domains with conical points. * The precise power modulus of continuity at singular boundary point for solutions of the Dirichlet, mixed and the Robin problems. * The behaviour of weak solutions near conical point for the Dirichlet problem for m – Laplacian. * The behaviour of weak solutions near a boundary edge for the Dirichlet and mixed problem for elliptic quasilinear equations with triple degeneration. * Precise exponents of the solution decreasing rate near boundary singular points and best possible conditions for this. * The question about the influence of the coefficients smoothness on the regularity of solutions. * New existence theorems for the Dirichlet problem for linear and quasilinear equations in domains with conical points. * The precise power modulus of continuity at singular boundary point for solutions of the Dirichlet, mixed and the Robin problems. * The behaviour of weak solutions near conical point for the Dirichlet problem for m - Laplacian. * The behaviour of weak solutions near a boundary edge for the Dirichlet and mixed problem for elliptic quasilinear equations with triple degeneration.

This book presents a unified theory of the Finite Element Method and the Boundary Element Method for a numerical solution of second order elliptic boundary value problems. This includes the solvability, stability, and error analysis as well as efficient methods to solve the resulting linear systems. Applications are the potential equation, the system of linear elastostatics and the Stokes system. While there are textbooks on the finite element method, this is one of the first books on Theory of Boundary Element Methods. It is suitable for self study and exercises are included.

Hierarchical matrices are an efficient framework for large-scale fully populated matrices arising, e.g., from the finite element discretization of solution operators of elliptic boundary value problems. In addition to storing such matrices, approximations of the usual matrix operations can be computed with logarithmic-linear complexity, which can be exploited to setup approximate preconditioners in an efficient and convenient way. Besides the algorithmic aspects of hierarchical matrices, the main aim of this book is to present their theoretical background. The book contains the existing approximation theory for elliptic problems including partial differential operators with nonsmooth coefficients. Furthermore, it presents in full detail the adaptive cross approximation method for the efficient treatment of integral operators with non-local kernel functions. The theory is supported by many numerical experiments from real applications.
The material of the present book has been used for graduate-level courses at the University of Ia~i during the past ten years. It is a revised version of a book which appeared in Romanian in 1993 with the Publishing House of the Romanian Academy. The book focuses on classical boundary value problems for the principal equations of mathematical physics: second order elliptic equations (the Poisson equations), heat equations and wave equations. The existence theory of second order elliptic boundary value problems was a great challenge for nineteenth century mathematics and its development was marked by two decisive steps. Undoubtedly, the first one was the Fredholm proof in 1900 of the existence of solutions to Dirichlet and Neumann problems, which represented a triumph of the classical theory of partial differential equations. The second step is due to S. 1. Sobolev (1937) who introduced the concept of weak solution in partial differential equations and inaugurated the modern theory of boundary value problems. The classical theory which is a product ofthe nineteenth century, is concerned with smooth (continuously differentiable) sollutions and its methods rely on classical analysis and in particular on potential theory. The modern theory concerns distributional (weak) solutions and relies on analysis of Sob ole v spaces and functional methods. The same distinction is valid for the boundary value problems associated with heat and wave equations. Both aspects of the theory are present in this book though it is not exhaustive in any sense.
This research monograph focusses on a large class of variational elliptic problems with mixed boundary conditions on domains with various corner singularities, edges, polyhedral vertices, cracks, slits. In a natural functional framework (ordinary Sobolev Hilbert spaces) Fredholm and semi-Fredholm properties of induced operators are completely characterized. By specially choosing the classes of operators and domains and the functional spaces used, precise and general results may be obtained on the smoothness and asymptotics of solutions. A new type of characteristic condition is introduced which involves the spectrum of associated operator pencils and some ideals of polynomials satisfying some boundary conditions on cones. The methods involve many perturbation arguments and a new use of Mellin transform. Basic knowledge about BVP on smooth domains in Sobolev spaces is the main prerequisite to the understanding of this book. Readers interested in the general theory of corner domains will find here a new basic theory (new approaches and results) as well as a synthesis of many already known results; those who need regularity conditions and descriptions of singularities for numerical analysis will find precise statements and also a means to obtain further one in many explicit situtations.
For the first time in the mathematical literature this two-volume work introduces a unified and general approach to the asymptotic analysis of elliptic boundary value problems in singularly perturbed domains. While the first volume is devoted to perturbations of the boundary near isolated singular points, the second volume treats singularities of the boundary in higher dimensions as well as nonlocal perturbations. At the core of this work are solutions of elliptic boundary value problems by asymptotic expansion in powers of a small parameter that characterizes the perturbation of the domain. In particular, it treats the important special cases of thin domains, domains with small cavities, inclusions or ligaments, rounded corners and edges, and problems with rapid oscillations of the boundary or the coefficients of the differential operator. The methods presented here capitalize on the theory of elliptic boundary value problems with nonsmooth boundary that has been developed in the past thirty years. Moreover, a study on the homogenization of differential and difference equations on periodic grids and lattices is given. Much attention is paid to concrete problems in mathematical physics, particularly elasticity theory and electrostatics. To a large extent the work is based on the authors' work and has no significant overlap with other books on the theory of elliptic boundary value problems.
For the first time in the mathematical literature, this two-volume work introduces a unified and general approach to the subject. To a large extent, the book is based on the authors’ work, and has no significant overlap with other books on the theory of elliptic boundary value problems
This is a monograph on the emerging branch of mathematical biophysics combining asymptotic analysis with numerical and stochastic methods to analyze partial differential equations arising in biological and physical sciences. In more detail, the book presents the analytic methods and tools for approximating solutions of mixed boundary value problems, with particular emphasis on the narrow escape problem. Informed throughout by real-world applications, the book includes topics such as the Fokker-Planck equation, boundary layer analysis, WKB approximation, applications of spectral theory, as well as recent results in narrow escape theory. Numerical and stochastic aspects, including mean first passage time and extreme statistics, are discussed in detail and relevant applications are presented in parallel with the theory. Including background on the classical asymptotic theory of differential equations, this book is written for scientists of various backgrounds interested in deriving solutions to real-world problems from first principles.
For the first time in the mathematical literature, this two-volume work introduces a unified and general approach to the subject. To a large extent, the book is based on the authors’ work, and has no significant overlap with other books on the theory of elliptic boundary value problems.
This self-contained textbook provides the basic, abstract tools used in nonlinear analysis and their applications to semilinear elliptic boundary value problems and displays how various approaches can easily be applied to a range of

model cases. Complete with a preliminary chapter, an appendix that includes further results on weak derivatives, and chapter-by-chapter exercises, this book is a practical text for an introductory course or seminar on nonlinear functional analysis.

The theory of nonlinear elliptic equations is currently one of the most actively developing branches of the theory of partial differential equations. This book investigates boundary value problems for nonlinear elliptic equations of arbitrary order. In addition to monotone operator methods, a broad range of applications of topological methods to nonlinear differential equations is presented: solvability, estimation of the number of solutions, and the branching of solutions of nonlinear equations. Skrypnik establishes, by various procedures, a priori estimates and the regularity of solutions of nonlinear elliptic equations of arbitrary order. Also covered are methods of homogenization of nonlinear elliptic problems in perforated domains. The book is suitable for use in graduate courses in differential equations and nonlinear functional analysis.

This 2000 book provided the first detailed exposition of the mathematical theory of boundary integral equations of the first kind on non-smooth domains.

A novel approach to analysing initial-boundary value problems for integrable partial differential equations (PDEs) in two dimensions, based on ideas of the inverse scattering transform that the author introduced in 1997. This method is unique in also yielding novel integral representations for linear PDEs. Several new developments are addressed in the book, including a new transform method for linear evolution equations on the half-line and on the finite interval; analytical inversion of certain integrals such as the attenuated Radon transform and the Dirichlet-to-Neumann map for a moving boundary; integral representations for linear boundary value problems; analytical and numerical methods for elliptic PDEs in a convex polygon; and integrable nonlinear PDEs. An epilogue provides a list of problems on which the author's new approach has been used, offers open problems, and gives a glimpse into how the method might be applied to problems in three dimensions.

Based on seven lecture series given by leading experts at a summer school at Peking University, in Beijing, in 1984. this book surveys recent developments in the areas of harmonic analysis most closely related to the theory of singular integrals, real-variable methods, and applications to several complex variables and partial differential equations. The different lecture series are closely interrelated; each contains a substantial amount of background material, as well as new results not previously published. The contributors to the volume are R. R. Coifman and Yves Meyer, Robert Fcfferman, Carlos K. Kenig, Steven G. Krantz, Alexander Nagel, E. M. Stein, and Stephen Wainger.

In recent years, there has been a great deal of activity in the study of boundary value problems with minimal smoothness assumptions on the coefficients or on the boundary of the domain in question. These problems are of interest both because of their theoretical importance and the implications for applications, and they have turned out to have profound and fascinating connections with many areas of analysis. Techniques from harmonic analysis have proved to be extremely useful in these studies, both as concrete tools in establishing theorems and as models which suggest what kind of result might be true. Kenig describes these developments and connections for the study of classical boundary value problems on Lipschitz domains and for the corresponding problems for second order elliptic equations in divergence form. He also points out many interesting problems in this area which remain open.

This EMS volume gives an overview of the modern theory of elliptic boundary value problems, with contributions focusing on differential elliptic boundary problems and their spectral properties, elliptic pseudodifferential operators, and general differential elliptic boundary value problems in domains with singularities.

Boundary Value Problems is a translation from the Russian of lectures given at Kazan and Rostov Universities, dealing with the theory of boundary value problems for analytic functions. The emphasis of the book is on the solution of singular integral equations with Cauchy and Hilbert kernels. Although the book treats the theory of boundary value problems, emphasis is on linear problems with one unknown function. The definition of the Cauchy type integral, examples, limiting values, behavior, and its principal value are explained. The Riemann boundary value problem is emphasized in considering the theory of boundary value problems of analytic functions. The book then analyzes the application of the Riemann boundary value problem as applied to singular integral equations with Cauchy kernel. A second fundamental boundary value problem of analytic functions is the Hilbert problem with a Hilbert kernel; the application of the Hilbert problem is also evaluated. The use of Sokhotski's formulas for certain integral analysis is explained and equations with logarithmic kernels and kernels with a weak power singularity are solved. The chapters in the book all end with some historical briefs, to give a background of the problem(s) discussed. The book will be very valuable to mathematicians, students, and professors in advanced mathematics and geometrical functions.

This book, which is a new edition of a book originally published in 1965, presents an introduction to the theory of higher-order elliptic boundary value problems. The book contains a detailed study of basic problems of the theory, such as the problem of existence and regularity of solutions of higher-order elliptic boundary value problems. It also contains a study of spectral properties of operators associated with elliptic boundary value problems. Weyl's law on the asymptotic distribution of eigenvalues is studied in great generality.

The theory of boundary value problems for elliptic systems of partial differential equations has many applications in mathematics and the physical sciences. The aim of this book is to "algebraize" the index theory by means of pseudo-differential operators and new methods in the spectral theory of matrix polynomials. This latter theory provides important tools that will enable the student to work efficiently with the principal symbols of the elliptic and boundary operators on the boundary. Because many new methods and results are introduced and used throughout the book, all the theorems are proved in detail, and the methods are well illustrated through numerous examples and exercises. This book is ideal for use in graduate level courses on partial differential equations, elliptic systems, pseudo-differential operators, and matrix

analysis.
The description for this book, Introduction to Partial Differential Equations. (MN-17), Volume 17, will be forthcoming.
Building on the basic techniques of separation of variables and Fourier series, the book presents the solution of boundary-value problems for basic partial differential equations: the heat equation, wave equation, and Laplace equation, considered in various standard coordinate systems--rectangular, cylindrical, and spherical. Each of the equations is derived in the three-dimensional context; the solutions are organized according to the geometry of the coordinate system, which makes the mathematics especially transparent. Bessel and Legendre functions are studied and used whenever appropriate throughout the text. The notions of steady-state solution of closely related stationary solutions are developed for the heat equation; applications to the study of heat flow in the earth are presented. The problem of the vibrating string is studied in detail both in the Fourier transform setting and from the viewpoint of the explicit representation (d'Alembert formula). Additional chapters include the numerical analysis of solutions and the method of Green's functions for solutions of partial differential equations. The exposition also includes asymptotic methods (Laplace transform and stationary phase). With more than 200 working examples and 700 exercises (more than 450 with answers), the book is suitable for an undergraduate course in partial differential equations.
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This book provides an elementary, accessible introduction for engineers and scientists to the concepts of ordinary and partial boundary value problems, acquainting readers with fundamental properties and with efficient methods of constructing solutions or satisfactory approximations. Discussions include: ordinary differential equations classical theory of partial differential equations Laplace and Poisson equations heat equation variational methods of solution of corresponding boundary value problems methods of solution for evolution partial differential equations The author presents special remarks for the mathematical reader, demonstrating the possibility of generalizations of obtained results and showing connections between them. For the non-mathematician, the author provides profound functional-analytical results without proofs and refers the reader to the literature when necessary. Solving Ordinary and Partial Boundary Value Problems in Science and Engineering contains essential functional analytical concepts, explaining its subject without excessive abstraction.
This volume endeavours to summarise all available data on the theorems on isomorphisms and their ever increasing number of possible applications. It deals with the theory of solvability in generalised functions of general boundary-value problems for elliptic equations. In the early sixties, Lions and Magenes, and Berezansky, Krein and Roitberg established the theorems on complete collection of isomorphisms. Further progress of the theory was connected with proving the theorem on complete collection of isomorphisms for new classes of problems, and hence with the development of new methods to prove these theorems. The theorems on isomorphisms were first established for elliptic equations with normal boundary conditions. However, after the Noetherian property of elliptic problems was proved without assuming the normality of the boundary expressions, this became the natural way to consider the problems of establishing the theorems on isomorphisms for general elliptic problems. The present author's method of solving this problem enabled proof of the theorem on complete collection of isomorphisms for the operators generated by elliptic boundary-value problems for general systems of equations. Audience: This monograph will be of interest to mathematicians whose work involves partial differential equations, functional analysis, operator theory and the mathematics of mechanics.
Perturbation of the boundary is a rather neglected topic in the study of PDEs for two main reasons. First, on the surface it appears trivial, merely a change of variables and an application of the chain rule. Second, carrying out such a change of variables frequently results in long and difficult calculations. In this book, first published in 2005, the author carefully discusses a calculus that allows the computational morass to be bypassed, and he goes on to develop more general forms of standard theorems, which help answer a wide range of problems involving boundary perturbations. Many examples are presented to demonstrate the usefulness of the author's approach, while on the other hand many tantalizing open questions remain. Anyone whose research involves PDEs will find something of interest in this book.
Elliptic Boundary Value Problems on Corner DomainsSmoothness and Asymptotics of SolutionsSpringer
This accessible monograph covers higher order linear and nonlinear elliptic boundary value problems in bounded domains, mainly with the biharmonic or poly-harmonic operator as leading principal part. It provides rapid access to recent results and references.
Diese Monographie spannt einen Bogen rund um die aktuelle Thematik Wavelets, um neueste Entwicklungen anhand aufeinander aufbauender Probleme darzustellen und das konzeptuelle Potenzial von Waveletmethoden für Partielle Differentialgleichungen zu demonstrieren.
The aim of the series is to present new and important developments in pure and applied mathematics. Well established in the community over two decades, it offers a large library of mathematics including several important classics. The volumes supply thorough and detailed expositions of the methods and ideas essential to the topics in question. In addition, they convey their relationships to other parts of mathematics. The series is addressed to advanced readers wishing to thoroughly study the topic. Editorial Board Lev Birbrair, Universidade Federal do Ceará, Fortaleza, Brasil Victor P. Maslov, Russian Academy of Sciences, Moscow, Russia Walter D. Neumann, Columbia University, New York, USA Markus J. Pflaum, University of Colorado, Boulder, USA Dierk Schleicher, Jacobs University, Bremen, Germany